

RESULT INSTABILITY IN SELECT SEARCH ENGINES: AN EXPERIMENT WITH TREND PROJECT ANALYSIS USING COMPOUND KEYWORD “COMPARATIVE LIBRARIANSHIP”

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ABSTRACT

The paper is an outcome of a research conducted on four search engines viz., Google, Yahoo, Bing, and Baidu evaluate the trend projection analysis in their results. The objectives were accompanied by a collection of series of data using the Compound keyword “Comparative Librarianship” from the field of Library and Information Science. A series of results were collected on a daily basis to project 50 days of the projected trend. The evaluation revealed that Bing shows a positive secular trend while Google, Yahoo! And Baidu shows a downward or negative secular trend. The instability is less in Google and Bing while Yahoo and Baidu show a tremendous instability in its search results.

KEYWORDS: *Trending, Search Engine, Fluctuation*

INTRODUCTION

From Encyclopedia to Digital libraries, from navigation to information sources, from the information explosion to chunks of information, the internet is always used as all -purpose tools in today’s digital era. Among all, search engines are primarily used as a first -hand reference tool for any query or allied matters (Fallows, 2004; Madden, 2003) the search engines differ in work, algorithm and the mechanism for query indexing etc (Sullivan, 2005). However, the outcome yielded from search engines varies in rank from a few thousand to even in millions because of the accessibility of interminable measure of data. Many reviews demonstrate that exclusive initial access of results are perused by the end users from a few pages on a normal search i.e., mere two pages with a default of 10 results for every page, a sum of 20 results at majority (Silverstein, Henzinger, Marais and Moricz, 1999; Spink, Ozmutlu, Ozmutlu and Jansen, 2002; Jansen and Spink, 2004; Jansen, Spink and Pedersen, 2005). The search engine result page determines the success of a search engine, Therefore result ranking holds utmost importance in this regard. Result indexing is merely based on term frequency and the inverse document frequency in case of classical Information Retrieval system (Baeza-Yates and Ribeiro-Neto, 1999). There are numerous parameters which are taken into account in indexing of web search result. In older IRS the top ranking parameter was: number of links pointing to a given web page (Brin and Page, 1998; Google, 2016), Other factors include the anchor text of the links pointing to the web page, the placement of the search terms in the document (terms occurring in title or header may get a higher weight), the distance between the search terms, popularity of the page (in terms of the number of times it is visited), the text Appearing in meta tags (Yahoo, 2017), authority of subject

specific of the web page (Kleinberg, 1999; Teoma, 2005), recently search index and exactness of the hits are included as well (MSN, 2005). Search engines are always in competition for the betterment of result ranking using data from web page authors and ranking pages respectively. This is the main reason search engines keep their algorithm secret. Search engine companies like Google, 2016 which states "Due to our area of interest and the nature of business the main aim is to protect the integrity of our search results, the only information we make available to the public is results made by our algorithm ranking system." The updating and upgrading of search engines is a routine process, Search engine continues to alter their algorithm in order to improve their ranking results at par. Search Engine Optimization industries continually design and redesign, web pages in order to enhance the probability of getting a top hit in an information retrieval system.

It can be concluded that the first few page results in SERP (Search Engine Result Page) have major chances of getting a better hit and likely to be visited by the users. In addition to the examination of changes over time for the top results related to a given query of a search engine search engine optimization is on top priority for all information systems like Google and Yahoo. The SEO development has created a tremendous insatiability in the results when a query is given (Payne, 2005). However, this result insatiability can be of good or bad transformation between users "visceral need" (a fuzzy view of the information problem in the user's mind) and the "compromised need" (the way the query is phrased taking into account the limitations of the search tool at hand) (Taylor, 2009). Fluctuation in a search engine related to a query can only be judged by the user, while some researchers claim that it is impractical due to the presence of a large number of documents related to a query and all of them can't be viewed by the user, hence for checking fluctuation certain mathematical forecasting can be induced (Gordon and Pathak, 1999; TREC, 2014).

RELATED LITERATURE

A thorough literature search reveals that most studies focus on content changes rather than changes in the chronology of retrieved ranks of the same documents. For instance, Fetterly, Manasse, Najork, and Wiener, (2003) scrolled 15 lakh pages weekly for 90 days & found that 65 Percent of pages remain the same. Ntoulas, Cho, and Olston, (2004) also supports the same study & discovered that source of change is the addition of newly created pages rather update in existing pages. While Adar, Teevan, Dumais, and Elsas, (2009) crawled 5 million web pages hourly for a month & found that 34% of the pages did not have any change.

The approach was the attribute towards the difference in the sampling of the document too a much higher frequencies of change. In contrast, our work focuses on the changes in the SERP of selected search engines.

Teevan, Adar, Jones and Potts, (2007) studied the behavior of re-finding of web search uses and concluded that 40 percent of queries are re-finding queries. They also showed the detrimental impact is due to rank changes in a search engine result. The users are less likely to re-click the results and take more time to do so when a previously clicked result changed its position. Building on this insight our work presents the quantification and comparison of search results time instability in major search engines, as well as a detailed analysis on various aspects of this instability is also provided. The instability of search engine is not new, in fact, Selberg and Etzioni, (2000) found instability of search results more than ten years ago. In their research they showed that top ten hits on a search engine result page are replaced after a month, a rise of 54% was taken into account. Our study is different from this account as we use continuous series of data over a period of 100 days taking the hit score form the search engine itself and latter through trend projection method our analysis include many varieties of instability as well as the correlating factors.

PROBLEM

WWW was created for a purpose, mostly to share information through direct and command driven systems. There was no concept of Graphical User Interface. Systems like Archie, Gopher, and Veronica were commanded driven. After the information explosion, this software didn't cope up and hence obsolete. New concepts like Boolean operators, proximity searching, wildcards, truncation etc came into existence to cope with the searching for information explosion. Search engines began to develop new techniques and sophistication from the scholar's perspective, but it didn't help so far as search engine index in a different way. A further search engine doesn't sift information from users' point of view, i.e., it retrieves information on a particular topic from aspects like an advertisement, news, information, entertainment marketing mixed with some research papers. The information society attempt to filter information purely for scholarly queries, but a topic of interest differs from scholars to entrepreneurs. Information retrieval system continues to alter the algorithm in terms of quality devoid of fluctuations and instability. Numerous methods can assist to investigation fluctuation in the algorithm like Mozcast, SERP Metrics (Search engine result page Metrics), SERPs.com, Algoroo and Forecasting (Ayres, 2014).

The present investigation attempts to evaluate the instability among select search engines in terms of fluctuation and predict trend projection using forecasting for future fluctuation.

OBJECTIVES

- To select search engines for the study.

There are countless numbers of search engines over the internet. Some are active while others are inactive, some are country bound other are global, some are subjective, unilingual, etc while others are general, multilingual etc. Selection of search engines will be based on the following parameters.

- Automatic Indexing.
- Global Coverage.
- Availability of Counter meter.
- To select a compound search term.

Since the scope of the study relates to the field of Library and Information Science. The term will be selected using classifying scheme for Library and Information Science and List of subject headings for narrow and broader refining. The terms will be further divided into three categories, i.e., Simple, Compound and Complex terms. Then in the compound term, a single term will be selected for the study.

- To collect data for 100 days.

After the selection of search engines and compound term, the Data will be collected from search engines in series for consistent 100 days.

- To compare trends by forecasting of time series analysis.

The 100 days data will be analyzed in a mathematical sequence called trend projection analysis and search engine

instability will be compared accordingly.

METHODOLOGY

The International Standard Organization (ISO) has certified 230 search engines (**Promote3.com, 2016**). Most users prefer robotic search engines as they allow the users to compose their own queries rather than simply follow pre specified search paths or hierarchy as in case of directories. Moreover, robotic search engines locate data in a similar way i.e., by the use of crawlers or worms. This distinguishing feature differentiates them from web directories like Yahoo! Where collections of links to retrieve URL's are created and maintained by subject experts, or by means of any automated indexing process. However, some of these services also include a robot driven search engine facility. But this is not their primary purposes. This due to this feature Yahoo! Was included in the study.

Meta search engine, e.g., Dogpile etc doesn't have their own database. These accesses the database of many robotic search engines simultaneously. Thus, these were excluded from the study.

Still, hundreds of robotic general search engines navigate the web, in order to limit the scope of study after preliminary study, following criteria was laid down for a selection of general search engines:-

- Availability of automated indexing
- Global coverage of data
- Quick response time.
- Availability of result counter

Following two general search engines were selected for the study for meeting all the criteria and being comprehensive in nature.

- Google
- Baidu

Since the study relates to the field of Library and Information Science but there is no specialized search engine in the subject so another specialized search engine which relates to the subject area i.e., Bing was taken for study. Thus, the search engines undertaken for evaluation of study are:-

- Google (General)
- Bing (Specific)
- Yahoo! (Directory)
- Baidu (Country Specific General Search engine)

SELECTION OF TERM

Selection of terms is not directly possible in diversified fields like Library and Information Science. Therefore, classification scheme like Dewey Decimal Classification was consulted to understand Broad/Narrow structure of Library and Information Science. It helped to get five general Fields i.e.,

- Information System
- Digital Library
- Library Automation
- Library Services
- Librarianship

These terms were then browsed in “LC list of subject Headings” which provided many other related terms (RT) and Narrow terms (NT). Further NT and RT attached to each other preferred or standard terms were also browsed which retrieved a large number of Library and Information Science terms. At first instance 140 Library and Information Science related terms were identified.

Some terms occurred more than once and duplication keywords were removed. It reduced the number to 100. Later terms were divided into three broad groups below:

- Application
- Transformation
- Interrelation

“Application” denotes utility of Library and Information science in various fields and about 50 terms came under this group. “Transformation” refers to a method of developing or manufacturing library services into practical market and 30 terms fall under this group. “Interrelation” means transformation/dependence of one subject onto another and 20 terms came under this group. Further, each category was sub-divided into groups.

“Application” into four i.e., “Reference service”, “Informatics”, “Information Retrieval” & “Information Sources”. “Transformation” into two i.e., “Digitization” & “Consortia”. “Interrelation” into two i.e., “Library Network” & “Information System”.

The terms in each group were arranged alphabetically and each term was given a tag. Later 19% of the terms were selected from each group using “Systematic Sampling” (i.e., the first item selected randomly and next item at specific intervals). It further reduced the number to 19. Finally, the selected terms were classified into three groups under “Simple”, “Compound” & “Complex Terms” (**Table:-1.0**). This was done in order to investigate how search engines control and handle simple and phrased terms.

“Simple Terms” containing a single word were submitted to the search engine in the natural form i.e., without punctuating marks. “Compound Terms” consisting of two words were submitted to the search engines in the form of phrases as suggested by respective search engines and “Complex Terms” composed of more than two words or phrases, were sent to the search engine with suitable Boolean operator “AND” & “OR” between the terms to perform special searches. From the compound terms the ⁿth Keyword via Random sampling “*Comparative Librarianship*” was taken for the study.

Table 1: Keywords

S. No	Compound Terms
1	Bibliometric Classification
2	Citation Analysis
3	<i>Comparative Librarianship</i>
4	Digital Preservation
5	Electronic Repositories
6	Library Automation
7	Semantic web

Testing Fluctuation (The Ups and Downs)

The result in a search engine may differ for the same keywords over a gap of time, as the documents on a web are continuously being altered both in terms of quantitative and qualitative procedures. This instability changes, i.e. both qualitative and quantitative are expressed as fluctuations. The change in quantity is expressed as “Result Fluctuation” and the change in qualitative are expressed as “Document” and “Indexing Fluctuation”. The instability or fluctuation may show increased or decrease in result or comprehensibility but its versatility depends on the algorithm it follows. The change can be of good (as removal of spam and useless pages) or bad (as crawlers don't get a chance to index an informative page).

A “Result Fluctuation” can be expressed when a search engine shows decrease or increase in a total number of retrieved results for a given query i.e., search results at two different intervals of time. In other words, the total number of results varies from two or many observations, e.g. if a query is searched in a search engine and retrieves say 1000 results, on the 2nd day the same query may show increase result fluctuation say 1050 or decrease rate fluctuation say 950. Both the fluctuation is termed as instability.

Secular Trending in Search Engine

The Trending is an estimate of a future event achieved by systematically combining and casting forward in a predetermined way from the data about the past. It is simply a statement about the future prediction. Trending is possible only when a history of data exists. The study collected 100 days of data samples from four search engines out of seven as the result-counter was available with Google, Bing, Yahoo and Baidu. The data collection was carried on 1st of Feb, 2017 and ended on 11th of May, 2017 collecting 100 samples for the keyword “*Comparative Librarianship*” in four search engines (**Annexure**).

In forecasting, process few points were taken into consideration as:

- 100 days of data sampling were taken into consideration (**Annexure**).
- As the data is seasonal, Trend Projection Method was taken into consideration.
- The results were taken from SERP (Search Engine Result Page) on a daily basis.
- A forecast of 50 days was generated (**Table:-2**).
- The results were evaluated on a scattered graph with a regression line

Trend Projection

Trending describes the instability in search engines. This instability can be traced in a time series forecasting depending on the trending line which meets in a series of previous data points and then projects the linear line into a future of both mediums to longrange forecasting. Our research has described the trending with line visually to a set of points on a mathematical graph. The graphical interpretation may differ as per historical data. The trending may however, be differentiated into three types:

- **Positive or Upward Secular Trend:** This trend describes the data into an Increasing/ Positive/ Upward manner, the database may increase in terms of versatility or comprehensiveness.
- **Negative/ Downward Secular Trend:** This trend describes the data into a decreasing/ Negative/ Downward manner, the database may decrease in terms of versatility or comprehensiveness.
- **Neutral or Straight Secular Trend:** This type of data is consistent with slight or no changes in the database of a search engine.

For the study, 400 samples from 4 search engines are taken into account to generate 200 results of projected data which are described in graphs.

The formula derived form the study is:-

$$t_t = b_0 + b_1 t$$

b_0 and b_1 can be derived as:

$$b_0 = \bar{y} - b_1 \bar{t}$$

$$b_1 = \frac{n \sum t y_t - \sum t \sum y_t}{n \sum t^2 - (\sum t)^2}$$

Where

t = days

y_t = Result of the search query

(Explained in Annexure)

The projected result **Table 2** , shows a vast fluctuation both in terms of positive Secular trend and negative secular trend. The estimate is given by a trending line in **Figure. 1, Figure: 2, Figure: 3 and Figure. 4.**

Table 2: Projected Data Using Trend Projection Method for 50 days for the Keyword “Reprints”

Days	Google	Bing	Yahoo!	Baidu
1	47631576	13179273	26491273	9148103
2	47668421	13205283	26326105	9143263
3	47708061	13234241	26156840	9139284
4	47748600	13264270	25983343	9135201
5	47790065	13293117	25805472	9131012
6	47832482	13322850	25620718	9126712
7	47878303	13343807	25433425	9124480
8	47927916	13364534	25241290	9109972
9	47979187	13384991	25044149	9107096
10	48029582	13410350	24841831	9104354
11	48081467	13433312	24631493	9106027
12	48132175	13456218	24415296	9108373
13	48186955	13479038	24193027	9113393
14	48246240	13501741	23964463	9119419
15	48307585	13527199	23729371	9126524
16	48365141	13552808	23484535	9138051
17	48421218	13581581	23226261	9143623
18	48478547	13610821	22959611	9150045
19	48530857	13637379	22684229	9157378
20	48586963	13660830	22399740	9165685
21	48647282	13690536	22105752	9175038
22	48705592	13717322	21801848	9182842
23	48758086	13744134	21484197	9191536
24	48831502	13767485	21169157	9208784
25	48907450	13793911	20843987	9227927
26	48968166	13823813	20497528	9234082
27	49029593	13861254	20138819	9237411
28	49099106	13899782	19767289	9240840
29	49162620	13943196	19378580	9237985
30	49223006	13995850	18971560	9234505
31	49287494	14051062	18548902	9222592
32	49352589	14101102	18113763	9209059
33	49418253	14149098	17665824	9208585
34	49492568	14198692	17200696	9207938
35	49560058	14254076	16713391	9200099
36	49644763	14311679	16198268	9185396
37	49723346	14371617	15670196	9168933
38	49803400	14434015	15116210	9150576
39	49889307	14499003	14543325	9125379
40	49972799	14566723	13946409	9097434
41	50062328	14641810	13324147	9066531
42	50158492	14715911	12666046	9037445
43	50248117	14793257	11987853	9005514
44	50339444	14874032	11275079	8970522
45	50451403	14963159	10544314	8943111
46	50567310	15061462	9776366	8895233
47	50697057	15160133	8983284	8864121
48	50822468	15268695	8144636	8808989
49	50962462	15383127	7276919	8770592
50	51123597	15508839	6369014	8710628

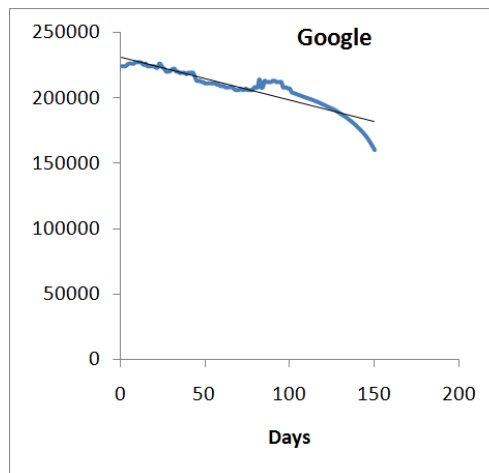


Figure 1: Negative Secular Trend of Google for the Keyword “Comparative Librarianship”

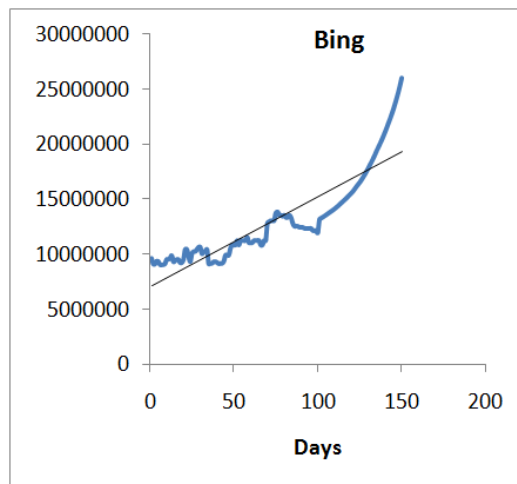


Figure 2: Positive Secular Trend of Google for the Keyword “Comparative Librarianship”

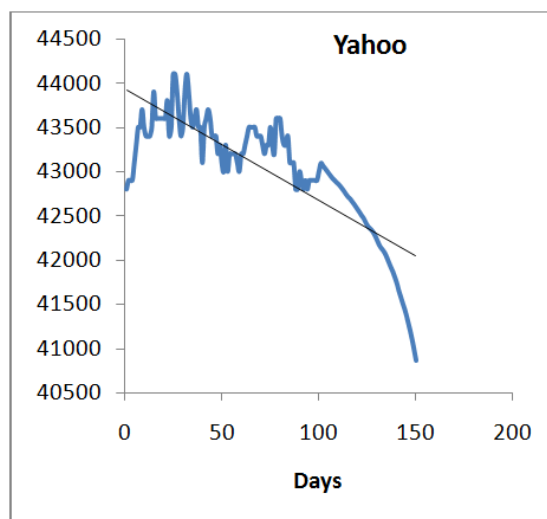


Figure 3: Negative Secular Trend of Google for the Keyword “Comparative Librarianship”

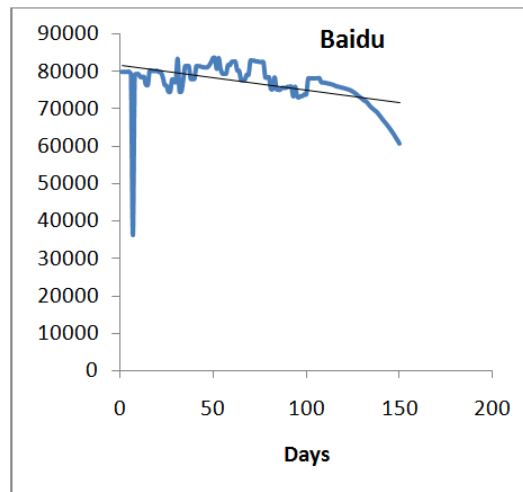


Figure 4: Negative Secular Trend of Google for the Keyword “Comparative Librarianship”

CONCLUSIONS

The trending of the search engines reveals that Bing shows a Positive secular trend while Google, Yahoo! And Baidu shows a negative or downward secular trending. The data forecasted to show a consistent growth in the database of Bing in terms of results. On the other hand Google, Yahoo! And Baidu shows down secular trending resulting in loss of database. As mentioned earlier the downward fluctuation can be of the good or bad case, depending on the algorithm, it follows e.g., removal of adware and spam. While positive, trending can be generalized as good as well as bad e.g., Recently, Google changed its algorithm to Panda and Penguin cause huge indexing fluctuation in Search Engine Result Page this is due the addition and deletion of newly created pages or removal of spam and adware. Baidu and Yahoo shows a tremendous instability in its database while Google and Bing shows minimal instability in its indexing result.

REFERENCES

1. Adar, E., Teevan, J., Dumais, S. T., & Elsas, J. L. (2009). *The Web Changes Everything: Understanding The Dynamics Of Web Content*. In: *Proceedings of the Second ACM International Conference on Web Search and Data Mining* (282–291). DOI: 10.1145/1498759.1498837
2. Ayres, K. (2014). *Understanding Why Search Rankings Fluctuate & How To Avoid A Ranking Demis*. Business2community. Retrieved from <http://www.business2community.com/seo/understanding-search-rankings-fluctuate-avoid-ranking-demise-0898216#vMk0Sl6gc41Bq6lC.97>
3. Baeza-Yates, R. A., & Ribeiro-Neto, B. A. (2004). *Modern Information Retrieval*. Facet Publishing.
4. Brin, S., & Page, L. (1998). *The Anatomy Of A Large-Scale Hypertextual Web Search Engine*. In: *Proceedings of the 7th International World Wide Web Conference, Computer Networks and ISDN Systems* (107 – 117). Available at: <http://www-db.stanford.edu/pub/papers/google.pdf>
5. Fallows, D (2004). *The Internet & Daily Life*. PEW Internet & American Life Project, Available at: http://www.pewinternet.org/pdfs/PIP_Internet_and_Daily_Life.pdf

6. Fetterly, D., Manasse, M., Najork, M., & Wiener, J. L. (2003). A Large-Scale Study Of The Evolution Of Web Pages. *In: Consortium of World Wide Web* (669–678). DOI: 10.1145/775152.775246
7. Google. (2016). *Google Information For Webmasters*, Google. Retrieved from <http://www.google.com/webmasters/4.html>
8. Gordon, M., & Pathak, P. (1999). Finding information of the World Wide Web: The retrieval effectiveness of search engines. *Information Processing and Management*, 35(5),141–180. DOI: 10.1016/S0306-4573(98)00041-7
9. Jansen, B. J., & Spink, A. (2004). An analysis of Web searching by European Alltheweb.Com Users. *Information Processing and Management*, 41(6), 361- 381. DOI: 10.1145/77556.77562
10. Jansen, B. J., Spink, A., & Pedersen, J. (2005). A Temporal Comparison Of Altavista Web Searching. *Journal of the American Society for Information Science and Technology*, 56(6), 559-570. DOI: 10.1002/asi.20145
11. Kleinberg, J. M. (1999). Authoritative Sources in A Hyperlinked Environment. *Journal of the ACM*, 46(5), 604-632. DOI: 10.1145/324133.324140
12. Madden, M. (2003). *America’s online pursuits: The changing picture of who’s online and what they do*. PEW Internet & American Life Project. Retrieved from http://www.pewinternet.org/pdfs/PIP_Online_Pursuits_Final.PDF
13. MSN Search. (2005). *Web search help: Change your search results by using Results Ranking*. MSN, Retrieved from http://search.msn.com/docs/help.aspx?t=SEARCH_PROC_BuildCustomizedSearch.htm
14. Ntoulas, A., Cho, J., & Olston, C. (2004). *What’s New On The Web? The Evolution Of The Web From A Search Engine Perspective*. *In: Consortium of World Wide Web* (1–12). ACM Press, 2004. DOI: 10.1145/155049.155074
15. Payne, C. (2016). *MSN Search launches*. MSN. Retrieved from <http://blogs.msdn.com/msnsearch/archive/2005/01/31/364278.aspx>
16. Promote3.com. (2016). *Top Search Engine Ranking Search Engine Optimization*. IDV International: California, Retrieved from <http://www.promote3.com/search-engine-230.htm>
17. Selberg, E., & Etzioni, O. (2000). *On The Instability Of Web Search Engines*. *In: Proceedings of RIAO* (223–235). Retrieved from <http://homes.cs.washington.edu/~etzioni/papers/riao2.pdf>
18. Peerzada Mohammad Iqbal et al., *Comprehensiveness, Dead Links and Duplicacy of Select Major Search Engines in the Field of Library and Information Science*, *International Journal of Library Science and Research (IJLSR)*, Volume 6, Issue 4, July-August 2016, pp. 1-10
19. Silverstein, C., Henzinger, M., Marais, H., & Moricz, M. (2010). *Analysis of a very large Web search engine query log*. *ACM SIGIR Forum*, 33(1) (2010), Retrieved from <http://www.acm.org/sigir/forum/F99/Silverstein.pdf>
20. Spink, S., Ozmutlu, O. H. C., & Jansen, B. J. (2002). *U.S. Versus European Web Searching Trends*. *SIGIR Forum*, Available at: <http://www.acm.org/sigir/forum/F2002/spink.pdf>

21. Sullivan, D. (2005). Nielsen Netratings Search Engine Ratings. *In: Proceedings of the 15th annual international ACM SIGIR conference on Research and development in information retrieval* (233-244). DOI: 10.1145/133160.133205
22. Taylor, R. S. (2009). Question-negotiation and information seeking in libraries. *College and Research Libraries*, 29, (178-194). DOI: 10.5860/crl.76.3.247
23. Teevan, J., Adar, E., Jones, R., & Potts, M. A. S. (2007). Information Re-Retrieval: Repeat Queries In Yahoo's Logs. *In: SIGIR* (151–158). DOI: 10.1145/1871437.1871662
24. Teoma (2005). Adding A New Dimension To Search: The Teoma Difference Is Authority. Teoma. Retrieved from <http://sp.teoma.com/docs/teoma/about/searchwithauthority.html>
25. Peerzada Mohammad Iqbal & Abdul Majid Baba, Currency of Research Articles for Select Major Search Engines in the Field of Library & Information Science, *IASET: International Journal of Library & Educational Science (IASET: IJLED)*, Volume 2, Issue 2, July-December 2016, pp. 61-70
26. TREC (2014). Data – English Relevance Judgements. TREC. Retrieved from http://trec.nist.gov/data/reljudge_eng.html
27. Yahoo. (2017). Yahoo! Help: How do I Improve the Ranking Of My Website In The Search Results. Yahoo. Retrieved from <http://help.yahoo.com/help/us/ysearch/ranking/ranking-02.html>

APPENDIX

Annexure:- Time Series Data For Forecasting of Select Search Engines For the Keyword “Comparative Librarianship”

Days (t)	Google			Bing			Yahoo!			Baidu		
	Result (Y _t)	Multiplication of Days and Results (tY _t)	Square of Days (t) ²	Result (Y _t)	Multiplication of Days and Results (tY _t)	Square of Days (t) ²	Result (Y _t)	Multiplication of Days and Results (tY _t)	Square of Days (t) ²	Result (Y _t)	Multiplication of Days and Results (tY _t)	Square of Days (t) ²
1	224000	224000	1	9690000	9690000	1	42800	42800	1	80000	80000	1
2	224000	448000	4	9170000	18340000	4	42900	85800	4	80000	160000	4
3	224000	672000	9	9170000	27510000	9	42900	128700	9	80000	240000	9
4	225000	900000	16	9450000	37720000	16	42900	171600	16	80000	320000	16
5	226000	1130000	25	9390000	46950000	25	43100	215500	25	80000	400000	25
6	226000	1356000	36	9120000	54720000	36	43300	259800	36	79200	475200	36
7	226000	1582000	49	9090000	63630000	49	43500	304500	49	36200	253400	49
8	226000	1808000	64	9110000	72880000	64	43500	348000	64	79200	633600	64
9	227000	2043000	81	9190000	82710000	81	43700	393300	81	79400	714600	81
10	227000	2270000	100	9590000	95900000	100	43500	435000	100	79400	794000	100
11	227000	2497000	121	9590000	105490000	121	43400	477400	121	78600	864600	121
12	227000	2724000	144	9720000	116640000	144	43400	520800	144	78600	943200	144
13	226000	2938000	169	9930000	129090000	169	43400	564200	169	78600	1021800	169
14	225000	3150000	196	9380000	131320000	196	43500	609000	196	76500	1071000	196
15	226000	3390000	225	9470000	142050000	225	43900	658500	225	76500	1147500	225
16	224000	3584000	256	9580000	153280000	256	43600	697600	256	80300	1284800	256
17	224000	3808000	289	9580000	162860000	289	43600	741200	289	80300	1365100	289
18	224000	4032000	324	9300000	167400000	324	43600	784800	324	80200	1443600	324
19	224000	4256000	361	9300000	176700000	361	43600	828400	361	80200	1523800	361
20	224000	4480000	400	9580000	191600000	400	43600	872000	400	80300	1606000	400
21	223000	4683000	441	10500000	220500000	441	43600	915600	441	79900	1677900	441
22	223000	4906000	484	10500000	231000000	484	43800	963600	484	79900	1757800	484
23	226000	5198000	529	9930000	228390000	529	43400	998200	529	78600	1807800	529
24	225000	5400000	576	9380000	225120000	576	43500	1044000	576	76500	1836000	576
25	223000	5575000	625	10100000	252500000	625	44100	1102500	625	76300	1907500	625
26	222000	5772000	676	10300000	267800000	676	44100	1146600	676	74700	1942200	676
27	220000	5940000	729	10300000	278100000	729	43900	1185300	729	74700	2016900	729
28	220000	6160000	784	10500000	294000000	784	43600	1220800	784	77900	2181200	784
29	220000	6380000	841	10700000	310300000	841	43400	1258600	841	77300	2241700	841
30	221000	6630000	900	10700000	321000000	900	43500	1305000	900	77300	2319000	900
31	222000	6882000	961	10100000	315100000	961	43900	1360900	961	83500	2588500	961
32	222000	7104000	1024	10300000	329600000	1024	44100	1411200	1024	74700	2390400	1024
33	220000	7260000	1089	10300000	339900000	1089	43900	1448700	1089	74700	2465100	1089
34	220000	7480000	1156	10500000	357000000	1156	43600	1482400	1156	77900	2648600	1156

35	219000	7665000	1225	9190000	321650000	1225	43500	1522500	1225	81500	2852500	1225
36	219000	7884000	1296	9230000	332280000	1296	43600	1569600	1296	81500	2934000	1296
37	219000	8103000	1369	9230000	341510000	1369	43700	1616900	1369	81500	3015500	1369
38	219000	8322000	1444	9380000	356440000	1444	43500	1653000	1444	78100	2967800	1444
39	218000	8502000	1521	9390000	366210000	1521	43500	1696500	1521	78100	3045900	1521
40	219000	8760000	1600	9310000	372400000	1600	43100	1724000	1600	78100	3124000	1600
41	219000	8979000	1681	9190000	376790000	1681	43500	1783500	1681	81500	3341500	1681
42	219000	9198000	1764	9230000	387660000	1764	43600	1831200	1764	81500	3423000	1764
43	219000	9417000	1849	9230000	396890000	1849	43700	1879100	1849	81500	3504500	1849
44	216000	9504000	1936	9410000	414040000	1936	43600	1918400	1936	81300	3577200	1936
45	213000	9585000	2025	9970000	448650000	2025	43400	1953000	2025	81200	3654000	2025
46	213000	9798000	2116	10000000	460000000	2116	43400	1996400	2116	81200	3735200	2116
47	213000	10011000	2209	9970000	468590000	2209	43400	2039800	2209	81200	3816400	2209
48	212000	10176000	2304	10600000	508800000	2304	43200	2073600	2304	81800	3926400	2304
49	212000	10388000	2401	10900000	534100000	2401	43300	2121700	2401	82600	4047400	2401
50	211000	10550000	2500	10900000	545000000	2500	43100	2155000	2500	83700	4185000	2500
51	211000	10761000	2601	10900000	555900000	2601	43000	2193000	2601	83700	4268700	2601
52	211000	10972000	2704	11300000	587600000	2704	43300	2251600	2704	80800	4201600	2704
53	211000	11183000	2809	10900000	577700000	2809	43000	2279000	2809	83700	4436100	2809
54	211000	11394000	2916	11300000	610200000	2916	43200	2332800	2916	80800	4363200	2916
55	211000	11605000	3025	11300000	621500000	3025	43200	2376000	3025	79500	4372500	3025
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57	210000	11970000	3249	11300000	644100000	3249	43200	2462400	3249	79500	4531500	3249
58	210000	12180000	3364	11600000	672800000	3364	43100	2499800	3364	81800	4744400	3364
59	209000	12331000	3481	11100000	654900000	3481	43000	2537000	3481	81800	4826200	3481
60	209000	12540000	3600	11100000	666000000	3600	43200	2592000	3600	82700	4962000	3600
61	209000	12749000	3721	11100000	677100000	3721	43200	2635200	3721	82700	5044700	3721
62	208000	12896000	3844	11300000	700600000	3844	43300	2684600	3844	82700	5127400	3844
63	208000	13104000	3969	11300000	711900000	3969	43400	2734200	3969	80300	5058900	3969
64	208000	13312000	4096	11300000	723200000	4096	43500	2784000	4096	80300	5139200	4096
65	208000	13520000	4225	11300000	734500000	4225	43500	2827500	4225	77800	5057000	4225
66	208000	13728000	4356	10900000	719400000	4356	43500	2871000	4356	77800	5134800	4356
67	207000	13869000	4489	10900000	730300000	4489	43500	2914500	4489	77800	5212600	4489
68	206000	14008000	4624	11300000	768400000	4624	43400	2951200	4624	79200	5385600	4624
69	206000	14214000	4761	11300000	779700000	4761	43400	2994600	4761	79200	5464800	4761
70	206000	14420000	4900	12900000	903000000	4900	43400	3038000	4900	83000	5810000	4900
71	207000	14697000	5041	13000000	923000000	5041	43300	3074300	5041	83000	5893000	5041
72	206000	14832000	5184	13100000	943200000	5184	43200	3110400	5184	83000	5976000	5184
73	206000	15038000	5329	13100000	956300000	5329	43300	3160900	5329	82800	6044400	5329
74	207000	15318000	5476	13100000	969400000	5476	43300	3204200	5476	82800	6127200	5476
75	206000	15505000	5625	13800000	1035000000	5625	43500	3262500	5625	82600	6195000	5625
76	206000	15656000	5776	13900000	1056400000	5776	43300	3290800	5776	82600	6277600	5776
77	206000	15862000	5929	13600000	1047200000	5929	43200	3326400	5929	82600	6360200	5929
78	206000	16068000	6084	13600000	1060800000	6084	43600	3400800	6084	78500	6123000	6084
79	208000	16432000	6241	13500000	1066500000	6241	43600	3444400	6241	78500	6201500	6241
80	208000	16640000	6400	13600000	1088000000	6400	43600	3488000	6400	78500	6280000	6400
81	208000	16848000	6561	13400000	1085400000	6561	43400	3515400	6561	75400	6107400	6561
82	214000	17548000	6724	13400000	1098800000	6724	43300	3550600	6724	75400	6182800	6724
83	208000	17264000	6889	13600000	1128800000	6889	43300	3593900	6889	78500	6515500	6889
84	208000	17472000	7056	13400000	1125600000	7056	43400	3645600	7056	75400	6333600	7056
85	213000	18105000	7225	12900000	1096500000	7225	43100	3663500	7225	75200	6392000	7225
86	212000	18232000	7396	12600000	1083600000	7396	43100	3706600	7396	75200	6467200	7396
87	212000	18444000	7569	12600000	1096200000	7569	43100	3749700	7569	75700	6585900	7569
88	212000	18656000	7744	12600000	1108800000	7744	42800	3766400	7744	75700	6661600	7744
89	212000	18868000	7921	12500000	1112500000	7921	42800	3809200	7921	75700	6737300	7921
90	213000	19170000	8100	12500000	1125000000	8100	43000	3870000	8100	76000	6840000	8100
91	213000	19383000	8281	12500000	1137500000	8281	42900	3903900	8281	76000	6916000	8281
92	212000	19504000	8464	12400000	1140800000	8464	42800	3937600	8464	76000	6992000	8464
93	212000	19716000	8649	12400000	1153200000	8649	42900	3989700	8649	73400	6826200	8649
94	212000	19928000	8836	12400000	1165600000	8836	42800	4032200	8836	76000	7144000	8836
95	212000	20140000	9025	12400000	1178000000	9025	42900	4075500	9025	73400	6973000	9025
96	208000	19968000	9216	12400000	1190400000	9216	42900	4118400	9216	73100	7017600	9216
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98	208000	20384000	9604	12200000	1195600000	9604	42900	4204200	9604	73500	7203000	9604
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100	207000	20700000	10000	12000000	1200000000	10000	43000	4300000	10000	73900	7390000	10000
Σt	$\Sigma(Y_t)$	ΣtY_t	$\Sigma(t)^2$	$\Sigma(Y_t)$	ΣtY_t	$\Sigma(t)^2$	$\Sigma(Y_t)$	ΣtY_t	$\Sigma(t)^2$	$\Sigma(Y_t)$	ΣtY_t	$\Sigma(t)^2$
5050	21528000	1069068000	338350	1100220000	59216700000	338350	4334800	218488600	338350	7854200	396084800	338350

